

**Report on the grant ONR N000 14-98-1-0070 "Modelling
Swell High Frequency Spectral and Wave Breaking"**

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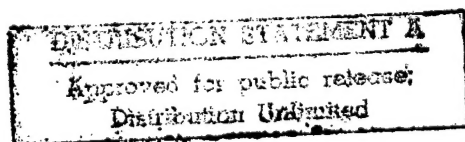
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1 Long-term goal

To develop an advanced physical model of the wind-generated spectra of gravity waves on the surface of deep water, paying special attention to universal spectra in the High Frequency region. To check the concept of weak-turbulent Kolmogorov spectra (Zakharov, Filonenko, 1968) by constructing angle-dependent analytical models of spectra and comparing them with the results of numerical simulation and laboratory experiments. To develop simplified models for evolution of energy-capacating part of spectra, including incoming swell. To construct a self-consistent theory of wave-breaking, making possible to give a well-justified expression for damping coefficient due to wave breaking.

2 Approach

Starting from the first principles (Navier-Stokes equations for water and air) we construct an array of models describing dynamics and kinetics of surface waves. The advanced methods of Hamiltonian formalism and conformal mapping are widely used. The most simplified models are solved numerically. The results are carefully compared with field and laboratory experiments, as well as with the results of a direct numerical solution of the kinetic equation.



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3 Work completed

By the use of the Hamiltonian formalism were recalculated three- and four-wave coupling coefficients on the surface of a fluid of arbitrary depth, and elaborate a self-consistent procedure for derivation of kinetic equation for waves in any physical situation. The hierarchy of simplified models for description of four-wave interaction was proposed. The most simple and suitable model based on the nonlinear diffusion equation in Fourier space was elaborated and studied analytically and numerically. A simplified model for description of narrow-angle spectra, like far-generated swell was studied analytically and numerically.

4 Results

Some of our results are really astonishing. A numerical simulation of a very simple and suitable model for four-wave interaction based on the use of the nonlinear diffusion equation gives almost as good results as a direct numerical simulation of the exact kinetic equation. Numerical solution of the diffusion equation takes much less of computer time and can be used in the next generation of the models of wave prediction (Zakharov, Pushkarev).

Direct numerical simulation of the exact kinetic equation confirms the concept of weak-turbulent Kolmogorov spectra and presence of persistent flux of energy and momentum to the region of high wave numbers (Perrie, Zakharov).

It was confirmed analytically that the four-wave coupling coefficient vanishes in the one-dimensional case. (This fact was found by computation by Dyachenko and Zakharov in 1994).

It was found that gravity waves on very shallow water are described by the integrable Kadomtsev-Petviashvili model. As a result, the terms of the leading order in the expansion powers of $1/kh$ cancel for the four-wave coupling coefficients. In spite of this fact, four-wave interaction on shallow water is *much stronger* than on deep water and is the dominant mechanism for interactions of small amplitude waves. At the same time, the conditions for weak-turbulent approach to wave-wave interaction on shallow water are very narrow and easily violated (Zakharov). In this case pure dynamical "three-wave" interaction takes the lead. A self-consistent statistical theory in this case is not developed so far.

5 Impact/application

The impact of this work will be most immediately felt in developing of weak-turbulent statistical theory on the finite depth water and in the progress in establishing of a universal form of spectra in high-frequency equilibrium region. The nonlinear diffusion model is very perspective from the practical view-point and should be seriously tested as a candidate for wave-prediction model of new generation.

6 Transitions

None yet.

7 Related projects

The ONR grant N00014-98-1-0439 FRS305900 "The response of wind ripples to long surface waves".

8 Publications

1. V.E.Zakharov. "Weakly nonlinear waves on the surface of an ideal finite depth fluid, Amer.Math.Soc.(2), Vol.182, (1998), 167-197.
2. V.E. Zakharov, A.N. Pushkarev "Nonlinear diffusion approximation for kinetic equation for gravity waves", submitted for publication in Journal of Physical Oceanology
3. W.Perrie, V.E.Zakharov, "The equilibrium range cascade of wind-generated waves", accepted for publication to the European Journal of Mechanics (B/Fluids).

| REPORT DOCUMENTATION PAGE | | | |
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| 13. ABSTRACT (Maximum 200 words) It was found that in the absence of external forcing and viscosity NDE has the same motion integrals as HE - wave action, energy and momentum. It was also found that NDE equation has asymptotic solutions describing Komogorov spectra of surface waves corresponding to given energy, momentum and wave fluxes. It was found by direct numerical simulation of dynamical equations for capillary waves that wave turbulence exited by long-wave forcing obeys Kolmogorov spectrum of surface elevations predicted by weak turbulence theory | | | |
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